Final Exam for Introduction to Logic

1. Let's start with some basic questions about logical concepts, etc. [1 point each]

(a) Each of our rules represents a form of valid reasoning. Only one of the following sentences correctly describes valid reasoning. Which one does so?

A. If the conclusion is false, the premises must be true.
B. If the premises are true, the conclusion must be true.
C. If the premises are true, the conclusion must be false.
D. If the conclusion is true, the premises must be true.

(b) It is possible for an argument to be sound but not valid.

A. True
B. False

(c) Proving which of the following sequents demonstrates a logical equivalence?

A. A&B } B&A
B. A>B, B>A } A<>B
C. } (AvB)<>~(~A&~B)
D. } (A>B)>(~B>~A)

(d) Which of the following is **not** a connective in Sentence Logic?

A. Only
B. If
C. And
D. Or

(e) "Batman is awesome and Batman is Bruce Wayne." The two instances of "is" in this sentence are

A. the 'is of identity' and the 'is of predication', respectively.
B. the 'is of predication' and the 'is of identity', respectively.
C. both the 'is of identity'.
D. both the 'is of predication'.

2. On to translations: Match each of the following formal translations with the appropriate sentence below. Note: You should use each of A-J only once. [0.5 points each]

A. Pb>(Ex)((Px&Wx)&~x=a)

B. Pa<>a=b

C. ~Pa>(x)Sx

D. ~Pa&~Cb

E. Pb>(x)(Px>(x=b v Wx))

F. Pa&(Ex)Sx

G. Pa v a=b

H. ~(Pa v Pb)

I. Pb>(x)((Px&Wx)>x=a)

J. Pa>~Pb

____ Alice is not going to the party, nor does Bill care.

____ Neither Alice nor Bill are going to the party.

____ Everyone will be sad unless Alice goes to the party.

____ Alice is going to the party, but some people are still sad.

____ Either Alice is going to the party or Alice is Bill.

____ Alice is going to the party if and only if she is Bill.

____ If Alice goes to the party, Bill won't.

____ Bill is going to the party only if Alice will be the only woman there.

____ Bill is going to the party only if Alice will not be the only woman there.

____ Bill is going to the party only if, besides himself, only women will be there.

3. Some special sequents: Please prove that each of the following is valid.

(a) The Law of Non Contradiction: [1 point]

} ~(P&~P)

(b) The Law of the Excluded Middle: [2 points]

} Pv~P

(c) There are two ways to translate "Neither… nor…" Please demonstrate that they are logically equivalent: [4 points]

} (~P&~Q)<>~(PvQ)

4. Ok, you got the long, hard one over and done with! Now just some regular old sequents. Please prove that each is valid.

(a) P>R, Q>(R&S) ~R } ~(PvQ) [2 points]

(b) P<>Q, PvQ } P&Q [2.5 points]

(c) (x)(Qx>Px), ~Pa } ~(x)Qx [1.5 points]

(d) (x)(Px>Qx), (Ex)Rx>a=b } Rc>(Pa>Qb) [2 points]